Enhanced FP1 Examination

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Rules

Allowed time: 1 hour 30 minutes.

Answer all questions, and any options that apply to you.

This is an enhanced examination paper for the further mathematics module *Further Pure Mathematics 1*. It is meant to be a significantly more challenging examination of general concepts specified by different boards. It is fully doable for students with knowledge of standard FP1, C1, and C2 material.

Unique question options have been included for students sitting examinations with certain boards.

Edexcel students: Answer questions 8, 9, 10, and 11.

MEI students: Answer questions 12, 13, 14, and 15.

1 Given that $\mathbf{M} = \begin{pmatrix} -1 & 0 \\ 2 & 1 \end{pmatrix}$, prove by induction that

$$\mathbf{M}^n = \begin{pmatrix} (-1)^n & 0 \\ 1-(-1)^n & 1 \end{pmatrix}$$

2 Express $\sqrt{2-7i} + \sqrt{1+i}$ in the form $a + bi$ where $a, b \in \mathbb{R}$.

3 Evaluate the sum

$$\sum_{r=0}^{n} [(3-r)^2 - (r+1)^2]$$

4 Sketch the locus $\arg(z+i) > \arg(z-1)$ on an Argand diagram.

5 The quintic polynomial

$$z^5 - 5z^4 + 7z^3 - 13z^2 + 60z - 50$$

has five roots, including 1 and $3+i$. Find the other three roots.

6 Let $\mathbf{A} = \begin{pmatrix} 3 & -1 \\ 2 & 0 \end{pmatrix}$.

(a) Show that $\mathbf{A}^2 - 3\mathbf{A} = -2\mathbf{I}$.

(b) Hence express $\mathbf{A}^{-1}$ in terms of $\mathbf{A}$ and $\mathbf{I}$.

(c) Verify your relation for $\mathbf{A}^{-1}$ in **(b)**, showing all your working.

7 Explain the graphical transformation given by $\begin{pmatrix} \frac{3}{\sqrt{2}} & -\frac{3}{\sqrt{2}} \\ \frac{3}{\sqrt{2}} & \frac{3}{\sqrt{2}} \end{pmatrix}$.

For Edexcel students

8 In this question $f(x) = x^{\frac{3}{4}} + 2x^{\frac{1}{2}} - 1$ and $x \geq 0$.

(a) Show that a root of $f(x)$ lies in the interval $[0.1, 0.2]$.

(b) If $f(\alpha) \approx 0$, use similar right-angle triangles to show that

$$\alpha = \frac{0.1f(0.2) + 0.2f(0.1)}{f(0.2) + f(0.1)}$$

(c) Hence find an approximate root of $f(x)$ to three decimal places.

9 Joe wants to solve $f(x) = 1 - \sqrt{2x}$ using the Newton-Raphson method.

(a) Explain why $x_0 = 0$ would not work as an initial estimate.

(b) If $x \geq 0$, use $f'(x)$ to show that $f(x)$ has only one root.

10 C is a curve with equation $y^2 = 4kx$ where $k \neq 0$ is a real constant.

(a) Find the points where C intersects the straight line $y = kx$.

P is a point on C with coordinates $(kp^2, 2kp)$.

(b) Show that the gradient of C at P is equal to $\frac{1}{p}$.

11 H is a curve with equation $xy = 1$.

(a) Sketch the curve H.

A straight line l has equation $y = kx$ where $k \neq 0$ is a real constant.

(b) Find the two points a and b where H intersects l.

(c) Hence find the shortest possible distance between a and b.

For MEI students

12 Let $f(x) = \frac{x+2}{x^3-8}$ where $-3 < x < 3$.

(a) Write down the equation of the vertical asymptote of $f(x)$.

(b) Sketch $f(x)$. Clearly mark all asymptotes and points of interest.

13 Let $f(x) = \frac{a-x}{ax+1}$ where a is an integer and $-2 < x < 2$.

(a) Find the points where $f(x)$ crosses the x and y axes in terms of a.

(b) Find the equation of the vertical asymptote of $f(x)$ in terms of a.

(c) Hence sketch $f(x)$ for $a = 4$.

(d) If $a \geq 1$, find the smallest value of x such that $f(x) \geq 1$.

14 The quadratic $ax^2 + bx + c$ has roots α and β.

(a) Form a new quadratic with the repeated root $\frac{1}{\alpha+\beta}$.

(b) Form a new quadratic with roots $\alpha - 1$ and $\beta - 1$.

15 You are given that $\frac{5}{r(r+5)} = \frac{1}{r} - \frac{1}{r+5}$.

(a) Evaluate the sum

$$\sum_{r=0}^{n} \frac{5}{r(r+5)}$$

(b) Hence find

$$\lim_{n\to\infty} \sum_{r=0}^{n} \frac{5}{r(r+5)}$$